<u>OPTICS</u>

RAY OPTICS AND OPTICAL INSTRUMENTS

Reflection

Laws of reflection

- The angle of reflection (i.e., the angle between reflected ray and the normal to the reflecting surface or the mirror) equals the angle of incidence (angle between incident ray and the normal).

$$\angle i = \angle r$$

- The incident ray, reflected ray and the normal to the reflecting surface at the point of incidence lie in the same plane.

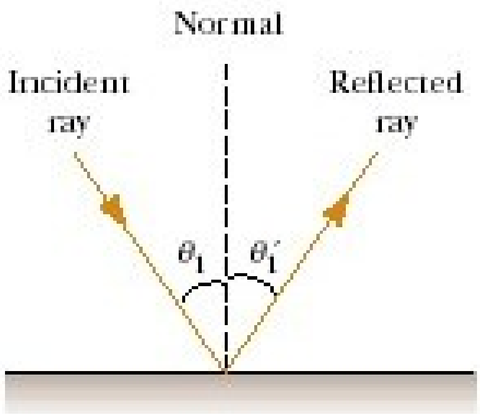
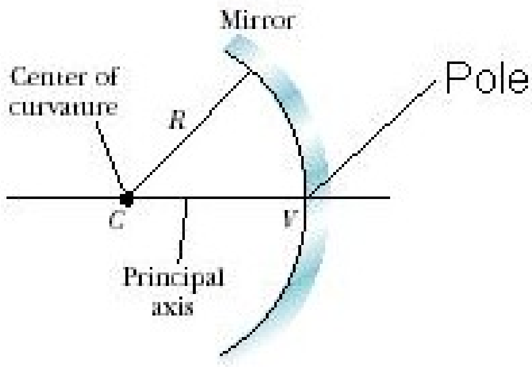


Figure 1: According to the laws of reflection,$\theta_1' = \theta_1$. The incident ray , reflected ray , and the normal all , lie in the same plane.

Spherical Mirrors
Pole: The geometric centre of a spherical mirror is called its pole.
Centre of curvature: The centre of a spherical shell of which the spherical mirror forms a section is called the centre of curvature of the mirror.***All points on the mirror lie at the same distance*** r ***from the centre of curvature***.

Principal Axis:.The line joining the pole and the centre of curvature of the spherical mirror is known as the principal axis.



Sign Convention:
Need for sign convention:A common accepted convention, makes it possible to have a single formula for spherical mirrors and a single formula for spherical lenses can handle all different cases.
Cartesian sign convention.

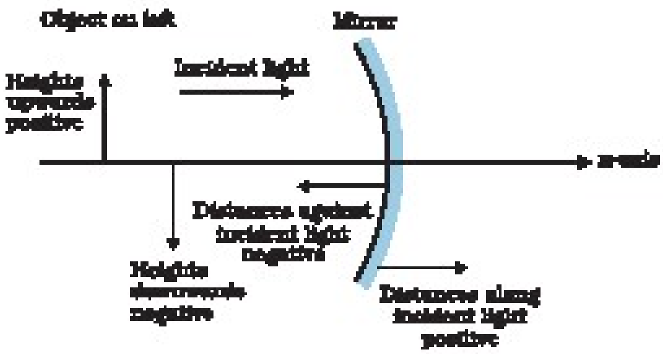


Figure 2: The Cartesian Sign Convention.

- ☞ All distances are measured from the pole of the mirror or the optical centre of the lens.(refer Fig.2)
- ☞ The distances measured in the same direction as the incident light are taken as positive.(refer Fig.2)
- ☞ Distances measured in the direction opposite to the direction of incident light are taken as negative.(refer Fig.2)
- ☞ The heights measured upwards with respect to x-axis and normal to the principal axis (x-axis) of the mirror/lens are taken as positive.(refer Fig.2)
- ☞ The heights measured downwards are taken as negative.(refer Fig.2)

The tabular column gives a clear picture of sign conventions for mirrors:-

QUANTITY	*NEGATIVE WHEN*	*POSITIVE WHEN*
Object distance (*u*)	Object is in front of mirror (real object)	Object is in back of mirror (virtual object)
Image distance (*v*)	Image is in front of mirror (real image)	Image is in back of mirror (virtual image)
Image height (*h'*)	Image is inverted	Image is upright
Focal length (*f*) and radius (*R*)	Mirror is concave	Mirror is convex
Magnification (*M*)	Image is inverted	Image is upright

Figure 3: sign convention for mirrors

Focal length of spherical mirrors

Focus and Focal length of Concave mirrors: The point **F** on the principal axis of a concave mirror [Fig.4]on which the reflected rays converge when a parallel beam of light is incident on a concave mirror, is called the focus and the distance between the pole and the focus is called the focal length f.

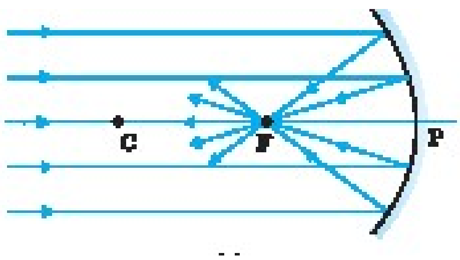


Figure 4: Focus-Concave mirror

Focus and Focal length of convex mirrors: For a convex mirror, the reflected rays appear to diverge from a point F on its principal axis when a parallel beam of light is incident[Fig.5]. The point F is called the principal focus of the mirror and the distance between the F and the pole is called the focal length f.

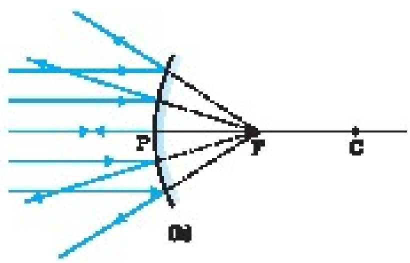


Figure 5: Focus-convex mirror

NOTE:It is assumed that the rays are **paraxial**, i.e., they are incident at points close to the pole

P of the mirror and make small angles with the principal axis.
Focal plane: If the parallel paraxial beam of light were incident, making some angle with the principal axis, the reflected rays would converge (or appear to diverge) from a point in a plane through F normal to the principal axis. This is called the focal plane of the mirror [Fig.6].

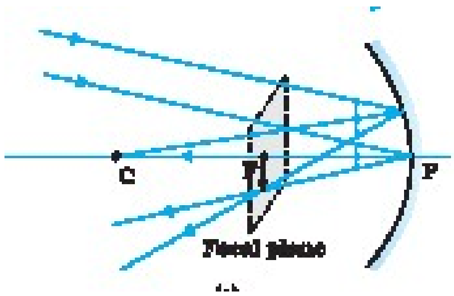


Figure 6: Focal plane

Relation between focal length and the radius of Curvature
Let C be the centre of curvature of the mirror and R the radius of curvature. Consider a ray

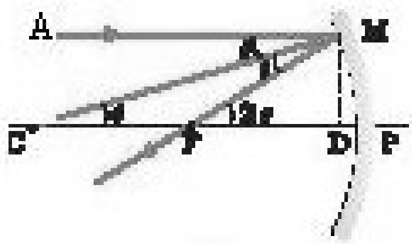


Figure 7: Geometry of reflection of an incident ray on a concave mirror

parallel to the principal axis striking the mirror at M. Then CM will be perpendicular to the mirror at M. Let θ be the angle of incidence, and MD be the perpendicular from M on the principal axis. Then from Fig.7, According to the laws of reflection

$$\angle AMC = \angle AMF \text{ and}$$

$$\angle AMC = \angle MCP \text{ (alternate angles)}$$

$\angle MFP$ is the external angle of $\triangle MCF$ hence

$$\angle MFD = 2\theta$$

In the $\triangle MCD$

$$tan\theta \approx \theta = \frac{MD}{CD} \quad (1)$$

In the $\triangle MFD$

$$tan2\theta \approx 2\theta = \frac{MD}{FD}$$
$$\theta = \frac{MD}{2FD} \quad (2)$$

equating (1)to (2)we get

$$\frac{MD}{CD} = \frac{MD}{2FD}$$

Or

$$CD = 2FD$$

As D is very close to P $CD \approx CP = R$ and $FD \approx FP = f$ the above equation becomes:-

$$f = \frac{R}{2} \tag{3}$$

For a concave lens as per the sign convention R is negative hence

$$f = \frac{-R}{2}$$

The above equation can be derived for a convex lens also, using the diagram shown below in Fig.

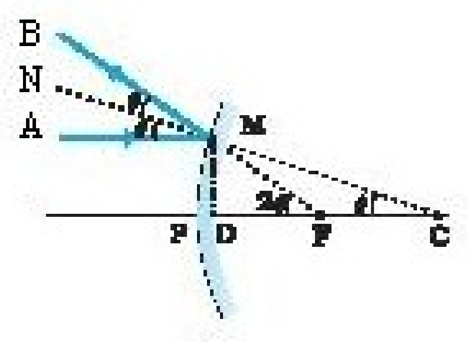


Figure 8: Geometry of reflection for convex mirror

Important:

1. The focal length of a mirror(whether concave or convex) depends only on the radius of curvature

2. for concave mirror the focal length is given by $f = \frac{-R}{2}$ and for convex mirror the focal length is given by $f = \frac{R}{2}$.

Ray Diagrams for Mirrors

The positions and sizes of images formed by mirrors can be conveniently determined with ray diagrams. These graphical constructions reveal the nature of the image and can be used to check results calculated from the mirror and magnification equations.To draw a ray diagram, we need to know the position of the object and the locations of the mirror mirrors focal point and center of curvature.We then draw three principal rays to locate the image,These rays all start from the same object point and are drawn as follows:-

For concave Mirror

1. **Ray 1** is drawn from the top of the object parallel to the principal axis and is reflected and then passes through the focus F.

2. **Ray 2** incident at any angle at the pole. The reflected ray follows laws of reflection($i = r$).

3. **Ray 3** is drawn from the top of the object through the focus and is reflected parallel to the principal axis.
4. **Ray 4** is drawn from the top of the object through the center of curvature C and is reflected back on itself(retraces itself).
5. The intersection of any two of these rays locates the image.
6. The other rays can be used as a check of the construction.

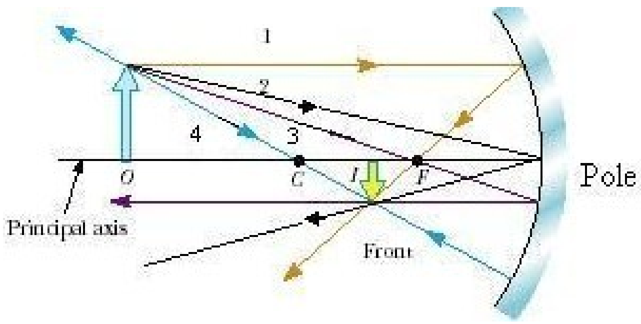


Figure 9: How to draw ray diagrams

For Convex Mirrors

1. **Ray 1** is drawn from the top of the object parallel to the principal axis and the reflected ray diverges,the extension of the reflected ray backward passes through the focus F.
2. **Ray 2** is drawn from the top of the object toward the focus on the back side of the mirror and is reflected parallel to the principal axis.
3. **Ray 3** is drawn from the top of the object toward the center of cur curvature C on the back side of the mirror and is reflected back on itself.
4. **Ray 4** incident at any angle at the pole. The reflected ray follows laws of reflection($i = r$)
5. **In a convex mirror mirror, the image of an object is always virtual, upright, and reduced in , size as shown in Figure 10.**

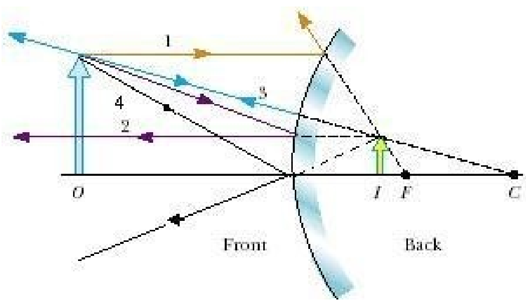


Figure 10: Drawing ray diagram for convex mirrors

The mirror Equation(Relation between$u$,v and f)
Figure.11 shows the ray diagram considering three rays(i) a ray parallel to the principle axis (ii) A

ray through the center of curvature and(iii) A ray reflected at the pole. It shows the image $A'B'$ (in this case, real) of an object AB formed by a concave mirror.The mirror equation or the relation between the object distance (u), image distance (v) and the focal length (f),can be *derived* as follows:-

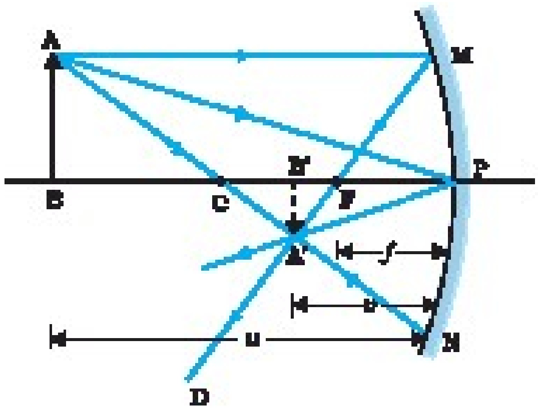


Figure 11: Ray Diagram for image formed by a concave mirror

NOTE: This derivation uses similar triangles ($A'B'F$ and MPF) and ($A'PB'$ and ABP) if you notice these triangles are chosen because the involve the physical quantities u, v, f, h, h'
From Fig.11, the two right-angled triangles $A'B'F$ and MPF are similar. (For paraxial rays, MP can be considered to be a straight line perpendicular to CP.) Therefore,

$$\frac{B'A'}{PM} = \frac{B'F}{FP}$$

Since $PM = BA$

$$\frac{B'A'}{BA} = \frac{B'F}{FP} \tag{4}$$

Since $\angle APB = \angle A'PB'$, the right angled triangles $A'PB'$ and ABP are also similar. Therefore,

$$\frac{B'A'}{BA} = \frac{B'P}{BP} \tag{5}$$

Comparing (4) and (5)we get

$$\frac{B'F}{FP} = \frac{B'P}{BP} \tag{6}$$

but $B'F = B'P - FP$ hence the above equation becomes

$$\frac{B'P - FP}{FP} = \frac{B'P}{BP} \tag{7}$$

Applying the sign conventions we get
$B'P = -v$,$FP = -f$ and $BP = -u$ using these equations in (7)we get

$$\frac{-v + f}{-f} = \frac{-v}{-u}$$

rearranging we get

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \tag{8}$$

Linear Magnification(m:is defined as the ratio of the height of the image (h') to the height of the object (h).

$$m = \frac{h'}{h}$$

as $h' = B'A'$ and $h = BA$ and from the similar right angled triangles $A'PB'$ and ABP.

$$m = \frac{h'}{h} = \frac{B'A'}{BA} = \frac{B'P}{BP} \tag{9}$$

applying sign conventions we get $B'P = -v$,$h' = -ve$,(because the image is inverted) and $BP = -u$

$$m = \frac{-h'}{h} = \frac{-v}{-u}$$

hence

$$m = -\frac{v}{u} \tag{10}$$

NOTE:The mirror equation and the magnification are valid for all mirrors convex or concave and all kinds of images virtual or real,if the appropriate sign conventions are used.

REFRACTION

Refraction: The direction of propagation of an obliquely incident ray of light that enters the other medium, changes at the interface of the two media. This phenomenon is called refraction of light.
Absolute Refractive Index(n):of a medium is defined as the ratio of the speed of light in vacuum to the speed of light in the medium.(light travels fastest in vacuum.)

$$n = \frac{speed\ of\ light\ in\ vacuum\,(c)}{speed\ oflight\ in\ a\ medium\,(v)} \tag{11}$$

or

$$n = \frac{c}{v} \tag{12}$$

Relative refractive index:when light travels from one medium of absolute refractive index (n_1)to another medium of refractive index(n_2)then the relative refractive index of the second medium with respect to the first is given by

$$n_{21} = \frac{speed\ of\ light\ in\ medium\,1\,(v_1)}{speed\ oflight\ in\ a\ medium\,2\,(v_2)} \tag{13}$$

or

$$n_{21} = \frac{v_1}{v_2} \tag{14}$$

Relation between n_{21},n_1 and n_2

Absolute refractive index of the first medium $n_1 = \frac{c}{v_1}$, where v_1 is the speed of light in the first medium,hence

$$v_1 = \frac{c}{n_1} \tag{15}$$

Absolute refractive index of the second medium $n_2 = \frac{c}{v_2}$, where v_2 is the speed of light in the second medium.hence

$$v_2 = \frac{c}{n_2} \tag{16}$$

using (15)and (16) in (14) we get

$$n_{21} = \frac{n_2}{n_1} \tag{17}$$

Relation between n_{21} and n_{12}

$$n_{12} = \frac{1}{n_{21}} \tag{18}$$

Laws of refraction

1. The incident ray, the refracted ray and the normal to the interface at the point of incidence, all lie in the same plane.

2. The ratio of the sine of the angle of incidence to the sine of angle of refraction is constant.i and r are the angles that the incident and its refracted ray make with the normal,respectively.Hence

$$\frac{sin\, i}{sin\, r} = n_{21} \tag{19}$$

where n_{21} is the refractive index of the second medium with respect to th first medium.This is called the **snell s law**.

Lateral Shift

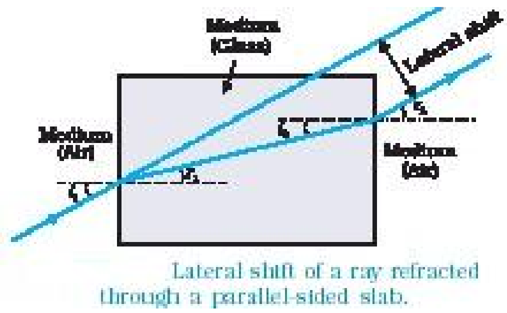


Figure 12:

- The angle of incidence is equal to the angle of emergence.*(proof left to reader)*

- For a rectangular slab, refraction takes place at two interfaces (air-glass and glass-air). It is easily seen from Fig. 12 that since $r_2 = i_1$, i.e., the emergent ray is parallel to the incident raythere is no deviation, but it does suffer lateral displacement/ shift with respect to the incident ray.

Refraction of light through multiple refracting media
If n_{32} is the refractive index of medium 3 with respect to medium 2 then

$$n_{32} = n_{31} \times n_{12}$$

where n_{31} is the refractive index of medium 3 with respect to medium 1. *(proof left to reader)*
Real Depth and apparent Depth Consider an Object(O) lying inside a denser medium(say

water) at a depth OA(Real depth),then due to refraction the object appears to be raised to a higher level i.e the image of the object appears closer to the eye now this depth O'A is called the **apparent depth**. The diagram below shows the ray diagram. Note that

$$\angle OAB = \angle OBN$$

(alternate interior angles)

$$\angle AO'B = r$$

(corresponding angles)

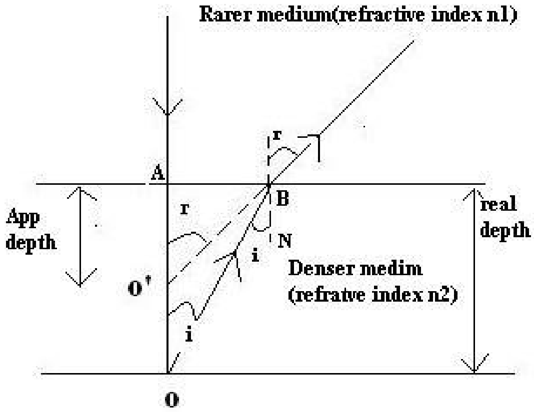


Figure 13: Real depth and apparent depth

In the $\triangle OAB$

$$sin\, i = \frac{AB}{OB}$$

In the $\triangle O'AB$

$$sin\, r = \frac{AB}{O'B}$$

Using the above equations in Snells law

$$\frac{sin\, i}{sin\, r} = n_{12}$$

$$\frac{AB/OB}{AB/O'B} = n_{12}$$

$$\frac{O'B}{OB} = n_{12}$$

since AB is small

$$OB \approx OA$$

(OA is real depth)

$$O'B \approx O'A$$

(O'B is apparent depth) applying the approximation

$$\frac{O'B\ (apparent\ depth)}{OA\ (real\ depth)} = n_{12}$$

OR

$$\frac{OA\ (real\ depth)}{O'B\ (apparent\ depth)} = n_{21}$$

NOTE: Real depth > apparent depth
Application of refraction: Among the myriad applications here is one interesting application:
The refraction of light through the atmosphere: The sun is visible a little before the actual sunrise and until a little after the actual sunset due to **refraction of light through the atmosphere** Fig.14).**Sunrise/sunset means the actual crossing of the horizon by the sun.** Fig 14 shows the actual and apparent positions of the sun with respect to the horizon. (The sun is in rarer medium(vacuum) and the observer is in the denser medium(atmosphere) . Due to this, the apparent shift in the direction of the sun by about half a degree and the corresponding time difference between actual sunset and apparent sunset is about 2 minutes .**(i.e even after the sun has set we see the sun for about 2 minutes and that sun is virtual!)** The apparent flattening (oval shape) of the sun at sunset and sunrise is also due to the same phenomenon. **NOTE**:See how

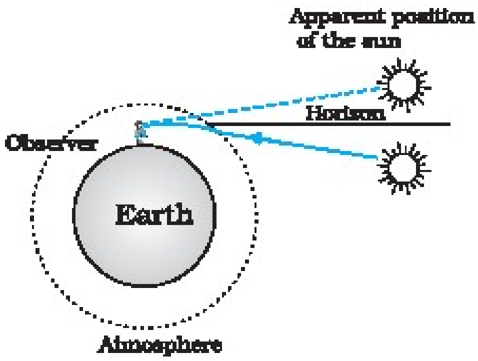
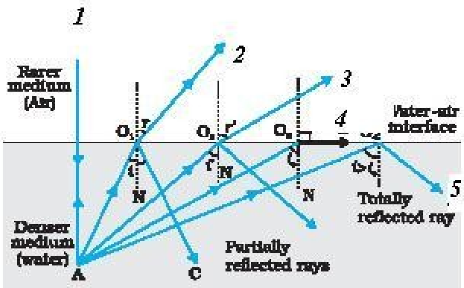


Figure 14: Apparent and real position of the sun due to refraction

the apparent image is drawn-the refracted ray is extended backward(dotted lines).
TOTAL INTERNAL REFLECTION: When light travels from an optically denser medium to a rarer medium at the interface, it is partly reflected back into the same medium and partly refracted to the second medium. This reflection is called the internal reflection.**At a particular angle of incidence i_c , called the critical angle, the refracted light ray moves parallel to the boundary(interface) so that $r = 90^o$ (Fig. 35.26b).At angles greater than the critical angle the the incident ray the incident light which comes from the denser medium is completely refracted back into the medium the is called *Total Internal Reflection.*** refraction.
Consider a light beam traveling in a denser medium and meeting the boundary between the two media , where n_2 is greater than n_1 (Fig.15). Various possible directions of the beam are indicated by rays 1 through 5. The refracted rays are bent away from the normal because n_2 is greater than n_1. At some particular angle of incidence , called the critical angle(i_c), the refracted light ray moves parallel to the boundary so that $r = 90^o$ (Fig.15).Applying Snell's at the interface :-

$$when\ i = i_c\ r = 90^o\ hence$$
$$\frac{sin\, i_c}{sin\, 90} = n_{12}$$
$$or\ sin\, i_c = n_{12}\ because\ sin\, 90 = 1$$



ray 1. The incident ray is normal to the surface suffers no refraction
ray2. & 3. As the angle of incidence increases the angle of refraction increases and bends farther away from the normal
ray 4. When the angle of incidence reaches the critical angle the refrated ray goes along the interface r=90
ray 5. When the incidence angle(i) is greater than the critical angle (ic)total internal reflection takes place

Figure 15: total internal reflection

$$or\ sin\, i_c = \frac{1}{n_{12}} = n_{21} \tag{20}$$

REMEMBER THESE

☞ Total internal reflection takes place ONLY when the incident ray travels from the denser medium to the rarer medium **AND** the angle of incidence is greater than the critical angle(conditions for total internal reflection)

☞ When light gets reflected by a surface, normally some fraction of it gets transmitted. The reflected ray, therefore, is always less intense than the incident ray, howsoever smooth the reflecting surface may be. In total internal reflection, on the other hand, no transmission of light takes place.

APPLICATIONS OF TOTAL INTERNAL REFLECTION
(you should know the details)

- Mirage
- Looming
- Fiber Optic cable
- Prism
- Diamond

TOTAL INTERNAL REFLECTION IN PRISM: Prisms are used to bend light by 90^0 or by 180^0 they make use of the principle of total internal reflection [Fig.16(a) and (b)]. Such a prism is also used to invert images without changing their size. **How they Work?** The angles of the prism are 45^o,45^o and 90^o if total internal reflection is to take place then the angle of incidence must be greater than the critical angle i.e the critical angle i_c for the material of the prism must be less than 45^o(only then i will be greater than i_c look at fig.16).(Using equan.(20)and the refractive index of crown glass(1.52) we get the critical angle as 41.14(if the material is Dense flint glass then $n = 1.62\ i_c = 37.31^0$)),hence total internal reflection takes place.

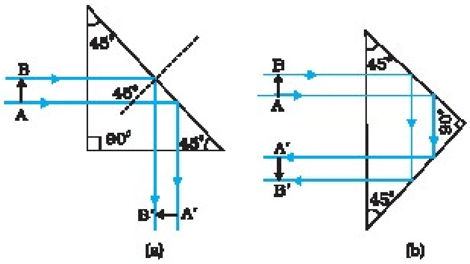


Figure 16: Total internal reflection in prism-note erect image and inverted image in (a) and (b)

REFRACTION AT SPHERICAL SURFACES AND BY LENSES

Sign Conventions for Refracting Surfaces

Quantity	Positive When	Negative When
Object location (u)	Object is in front of surface (real object)	Object is in back of surface (virtual object)
Image location (v)	Image is in back of surface (real image)	Image is in front of surface (virtual image)
Image height (h')	Image is upright	Image is inverted
Radius (R)	Center of curvature is in back of surface	Center of curvature is in front of surface

Figure 17: sign convention for curved spherical surfaces

REFRACTION AT A CONVEX REFRACTING SURFACE(object in the *rarer* medium):

Consider a convex refracting surface of refractive whose material has a refractive index n_2,radius of curvature R and center of curvature C. Fig.18 shows the geometry of formation of image I of an object O on the principal axis of a spherical surface. The rays are incident from a medium of refractive index n_1.

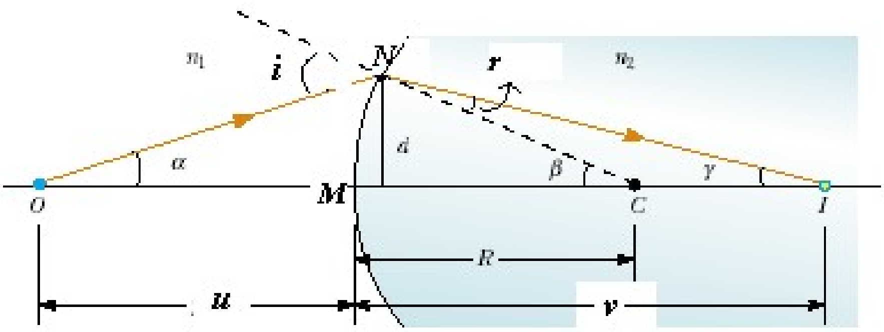


Figure 18: refraction by a convex refracting surface

Using the approximation that the aperture (or the lateral size) of the surface to be small,that NM will be taken to be nearly equal to the length of the perpendicular from the point N on the principal

axis.
Using small angle approximation

$$tan\,\alpha \approx \alpha = \frac{NM}{OM} \tag{21}$$

$$tan\,\beta \approx \beta = \frac{NM}{MC} \tag{22}$$

$$tan\,\gamma \approx \gamma = \frac{NM}{MI} \tag{23}$$

i is the external angle for the $\triangle NOC$ in fig.18 hence

$$i = \alpha + \beta$$

using equations (21) and (22)

$$i = \frac{NM}{OM} + \frac{NM}{MC} \tag{24}$$

similarly in the $\triangle NCI$ β is the external angle

$$\beta = r + \gamma \;\; or$$

$$r = \beta - \gamma \tag{25}$$

Using equations (22) and (23)

$$r = \frac{NM}{MC} - \frac{NM}{MI} \tag{26}$$

By Snell's law

$$\frac{sin\,i}{sin\,r} = n_{21} = \frac{n_2}{n_1}$$

$$or \;\; n_1 sin\,i = n_2 sin\,r$$

using small angle approximation

$$n_1 i = n_2 r$$

Using equans (24) and (26) for i and $r$we get

$$n_1\left(\frac{NM}{OM} + \frac{NM}{MC}\right) = n_2\left(\frac{NM}{MC} - \frac{NM}{MI}\right) \tag{27}$$

canceling NM we get

$$\left(\frac{n_1}{OM} + \frac{n_1}{MC}\right) = \left(\frac{n_2}{MC} - \frac{n_2}{MI}\right) \tag{28}$$

applying sign conventions

$$OM = -u, \;\; MI = +v, \;\; MC = +R$$

and substituting in (28)and rearranging we get

$$\left(\frac{n_2}{v} - \frac{n_1}{u}\right) = \left(\frac{n_2 - n_1}{R}\right) \tag{29}$$

REFRACTION AT A CONVEX REFRACTING SURFACE(object in the *denser* medium): Consider a convex refracting surface of refractive whose material has a refractive index n_2,radius of curvature R and center of curvature C. Fig.19 shows the geometry of formation of

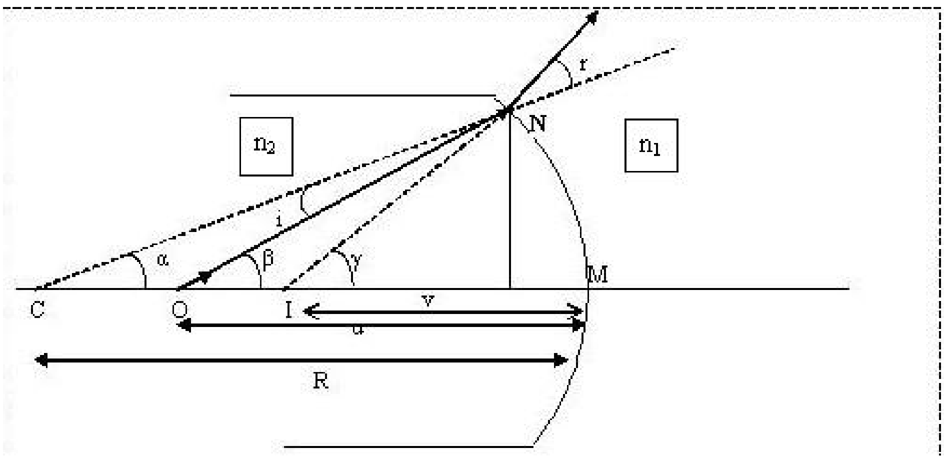


Figure 19: convex refracting surface with object inside the denser medium-notice the refracted ray bending away from normal

image I of an object O on the principal axis of a spherical surface. ***The rays are incident from a medium of refractive index*** n_2
Notice that the image is formed inside the denser medium and the image is virtual using small angle approximation

$$tan\,\alpha \approx \alpha = \frac{NM}{CM} \tag{30}$$
$$tan\,\beta \approx \beta = \frac{NM}{OM} \tag{31}$$
$$tan\,\gamma \approx \gamma = \frac{NM}{IM} \tag{32}$$

β is the external angle for the $\triangle NOC$ in fig.19 hence

$$\beta = \alpha + i \;\; hence$$
$$i = \beta - \alpha \tag{33}$$

similarly in the $\triangle NCI$ γ is the external angle

$$\gamma = r + \alpha \;\; or$$

($\angle CNI = r$(vertically opposite angles))

$$r = \gamma - \alpha \tag{34}$$

By Snell's law

$$\frac{sin\,i}{sin\,r} = n_{12} = \frac{n_1}{n_2}$$
$$or \;\; n_2 sin\,i = n_1 sin\,r$$

using small angle approximation

$$n_2 i = n_1 r$$

Using equans (33) and (34) for i and $r$we get

$$n_2\left(\frac{NM}{OM} - \frac{NM}{CM}\right) = n_1\left(\frac{NM}{IM} - \frac{NM}{CM}\right) \tag{35}$$

canceling NM we get

$$\left(\frac{n_2}{OM} - \frac{n_2}{CM}\right) = \left(\frac{n_1}{IM} - \frac{n_1}{CM}\right) \tag{36}$$

applying sign conventions

$$OM = -u, \;\; IM = -v, \;\; CM = -R$$

and substituting in (28)and rearranging we get

$$\left(\frac{n_1}{v} - \frac{n_2}{u}\right) = \left(\frac{n_1 - n_2}{R}\right) \tag{37}$$

NOTE:If you compare equations (37)and(29)you will notice that n_2 and n_1 have interchanged

LENS

A lens is a piece of glass or other transparent material which is bound by two spherical surfaces.The lensmakers formula gives the relationship between the radius of curvature of the bounding spherical surfaces,(r_1, r_2)(fig.20),focal length,object distance u, image distance(v) and the refractive index of the material of the lens.

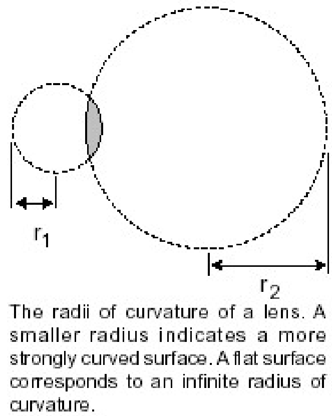


Figure 20: The shaded part represents the lens

LENS MAKERS FORMULA

Consider the lens shown in fig.21 below.ABC and ADC form the two spherical surface bounding the lens.R_1 is the radius of curvature of ADC and R_2 radius of curvature of ABC.

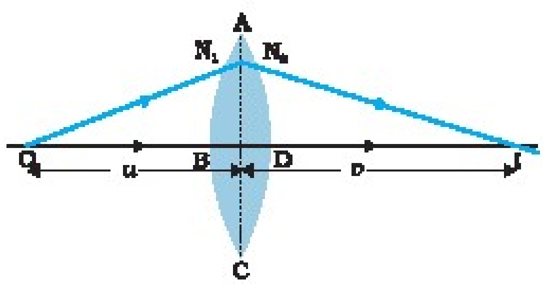


Figure 21: Formation of image by a lens

Consider the surface ABC shown in fig.22

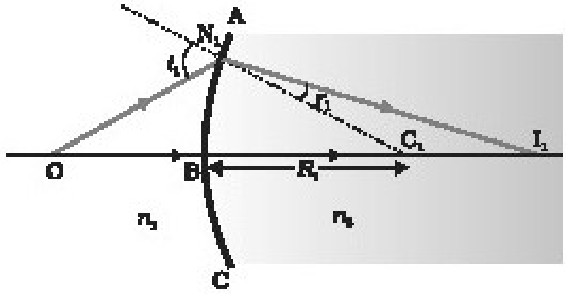


Figure 22: Refraction by the surface ABC

The first refracting surface forms the image I_1 of the object O. [Fig.22]. Applying(29) to the surface ABC with sign convention $u = -OB, R = +BC_1 \ andv = +BI_1$ we get Using equan.(29)

$$\left(\frac{n_2}{v} - \frac{n_1}{u}\right) = \left(\frac{n_2 - n_1}{R}\right)$$

$$\left(\frac{n_2}{BI_1} + \frac{n_1}{OB}\right) = \left(\frac{n_2 - n_1}{BC_1}\right) \tag{38}$$

Consider the surface ADC, ***the image I_1 acts as a virtual object for the second surface ADC.*** Applying(29) to the surface ADC with sign convention $u = +DI_1, R = -DC_2 \ andv = +DI$

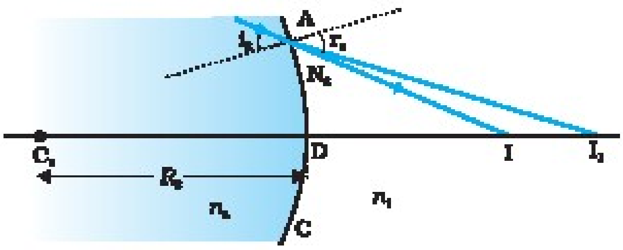


Figure 23: Refraction at the second spherical surface.

we get Using equan.(37) (interchange n_1 and n_2 of (29)to get (37))

$$\left(\frac{n_1}{v} - \frac{n_2}{u}\right) = \left(\frac{n_1 - n_2}{R}\right)$$

$$\left(\frac{n_1}{DI} - \frac{n_2}{DI_1}\right) = \left(-\frac{(n_1 - n_2)}{DC_2}\right) \tag{39}$$

adding (38) and (39) we get

$$\left(\frac{n_2}{BI_1} + \frac{n_1}{OB}\right) + \left(\frac{n_1}{DI} - \frac{n_2}{DI_1}\right) = \left(\frac{n_2 - n_1}{BC_1}\right) + \left(-\frac{(n_1 - n_2)}{DC_2}\right) \tag{40}$$

Since the lens is thin $BI_1 \approx DI_1$(i.e AD is negligibly small) the above equation can be approximated as

$$\frac{n_1}{OB} + \frac{n_1}{DI} = \frac{n_2 - n_1}{BC_1} + \frac{n_2 - n_1}{DC_2}$$

$$= \frac{n_1}{OB} + \frac{n_1}{DI} = (n_2 - n_1)\left(\frac{1}{BC_1} + \frac{1}{DC_2}\right) \tag{41}$$

Suppose the object is at infinity, i.e.,$OB = \infty$ $then$ $DI = f$,(if the object is at infinity then the image is formed at the focus) equan.(41) gives

$$= \frac{n_1}{f} = (n_2 - n_1)\left(\frac{1}{BC_1} + \frac{1}{DC_2}\right) \tag{42}$$

(because $\frac{n_1}{OB} = 0, ifOB = \infty$) By the sign convention,and using them in (42)

$$BC1 = +R1, DC2 = -R2$$

$$\frac{1}{f} = (\frac{n_2}{n_1} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

which can be written as

$$\frac{1}{f} = (n_{21} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \tag{43}$$

(because$\frac{n_2}{n_1} = n_{21}$) Equation (43) is known as the lens makers formula. It is useful to design lenses of desired focal length using surfaces suitable radii of curvature. **NOTE:** The formula is true for a concave lens also. In that case $R_1$is negative, R_2 positive and therefore, f is negative.

THE LENS EQUATION

Comparing Eqs.(41) and (42), we get (R.H.S equal so L.H.S equal)

$$\frac{n_1}{f} = \frac{n_1}{OB} + \frac{n_1}{DI}$$

In the thin lens approximation, B and D are both close to the optical centre of the lens. Applying the sign convention,

$$BO = -u, DI = +v$$

we get

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \tag{44}$$

NOTE: This formula is valid for both convex as well as concave lenses and for both real and virtual images (that's the advantage of sign convention!).

RAY TRACING FOR LENSES

For the purposes of ray tracing, every lens is said to have two focal points,first focal point (F_1) and a second focal point (F_2) . A converging lens has its first focal point on the side from where the light is coming , and the secondary focal point is symmetrically on the other side of the lens. The opposite is true of a diverging lens.As with mirrors, we can locate an image formed by a lens graphically, with the help of three rays (see Figures 24 and 25):

1. A ray parallel to the axis passes through (or appears to pass through-concave lens) the secondary focal point F2. (Ray 1 in the figures).

2. A ray passing through (or when extended, appearing to pass through-concave lens) the primary focal point F1 emerges from the lens parallel to the axis. (Ray 2 in the figures).

3. A ray falling on the lens at its centre passes through undeflected. (Ray 3 in the figures).

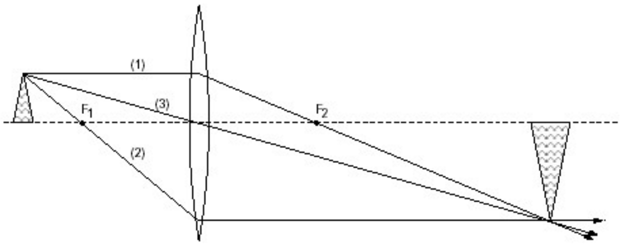


Figure 24: Ray tracing Convex Lens

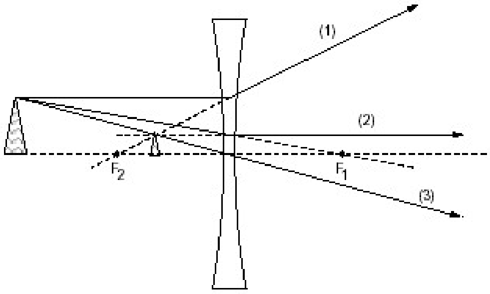


Figure 25: Ray tracing concave lens

Magnification
Magnification of a Lens is given by

$$m = \frac{h'}{h} = \frac{v}{u} \tag{45}$$

When we apply the sign convention, we see that, for erect (and virtual) image formed by a convex or concave lens, m is positive, while for an inverted (and real) image, m is negative.

APPLYING LENS MAKERS FORMULA

- Remember this diagram

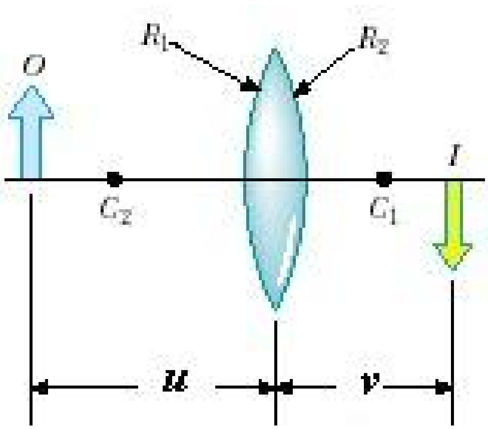


Figure 26: Lens Simplified

- Different types of lens:-

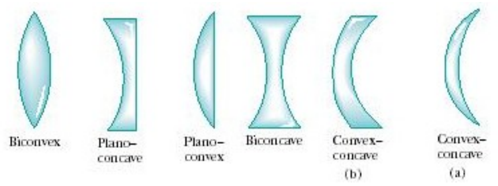


Figure 27: Differant types of lenses

- If one side of a lens is flat as in the case of Plano covex lens then that side has a radius of curvature $R = \infty$
- A lens has a Focus(F) on each side, front and back. However,there is only one focal length.Each of the two focal points is located the same distance from the lens . This can be seen mathematically by interchanging changing R1 and R2 in Equation (and changing the signs of the radii because back and front have been interchanged). As a result, the lens forms an image of an object at the same point if it is turned around.(i.e)***By turning the lens around the focal length does not change.***
- If you get confused between the mirror equation($\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$) and the lens equation($\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$) remember "Lens is less, mirror is more"

POWER OF A LENS:

Power is defined as the inverse of the focal length of a lens.

$$P = \frac{1}{f}$$

KEY POINTS

- Power of a lens is a measure of the convergence or divergence, which a lens introduces in the light falling on it.
- A lens with higher power bends the incident light more or in other words a lens of shorter focal length bends the incident light more.
- The unit of power is diopter,it is the inverse of focal length expressed in ***meters***. $1D = 1m^{-1}$

COMBINATION OF LENSES

If two thin lenses are used to form an image, the system can be treated in the following manner. First, the image formed by the first lens(I_1) is located as if the second lens were not present. Then a ray diagram is drawn for the second lens, with the image formed by the first lens now serving as the object for the second lens. The second image formed is the final image(I) of the system. If the image formed by the first lens lies on the back side of the second lens, then that image is treated as a virtual object for the second lens.

For the image formed by the first lens A, we get

$$\frac{1}{f} = \frac{1}{v_1} - \frac{1}{u} \tag{46}$$

For the image formed by the second lens B, we get

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{v_1} \tag{47}$$

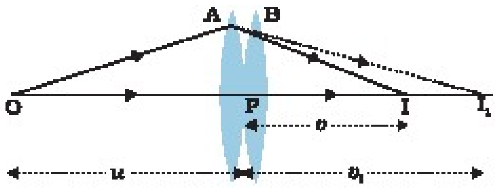


Figure 28: Lens Combination

Adding Eqs. (46)and (47), we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_1} \tag{48}$$

so that we get

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \tag{49}$$

Where f is the focal length of the combination. ***Therefore, two thin lenses in contact with each other are equivalent to a single thin lens having a focal length*** f ***given by Equation***(49)

REFRACTION THROUGH A PRISM

The prism Equation

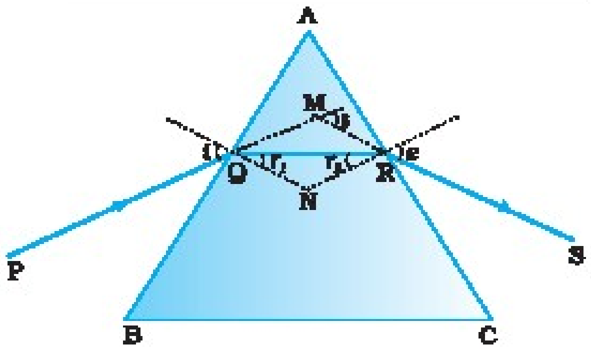


Figure 29: Refraction through a prism

Figure 29 shows the passage of light through a triangular prism ABC.

- The angles of incidence and refraction at the **first** face AB are i and r_1.
- The angle of incidence (from glass to air) at the second face AC is r_2
- The angle of refraction or emergence e
- A is the angle of the prism
- ***The angle between the emergent ray RS and the direction of the incident ray PQ is called the angle of deviation,*** δ

In the quadrilateral AQNR, two of the angles (at the vertices Q and R) are right angles.Therefore, the sum of the other angles of the quadrilateral is 180.

$$\angle A + \angle QNR = 180^o$$

From the triangle QNR,

$$r_1 + r_2 + \angle QNR = 180^o$$

equating the above two equations (RHS equal LHS equal)and canceling $\angle QNR$

$$\angle A + \angle QNR = r_1 + r_2 + \angle QNR \quad hence$$

$$r_1 + r_2 = A \tag{50}$$

$$\angle MQR = i - r_1$$

$$\angle MRQ = e - r_2$$

In the $\triangle MQR$, δ is the external angle hence

$$\delta = \angle MQR + \angle MRQ = i - r_1 + e - r_2 \quad hence$$

$$\delta = (i + e) - (r_1 + r_2)$$

Using equation (50)

$$\delta = (i + e) - A \tag{51}$$

A plot between the angle of deviation and angle of incidence is shown in Fig.30. The Plot shows that

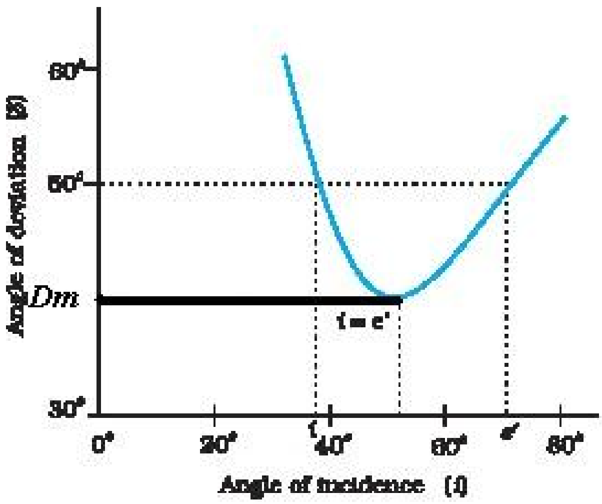


Figure 30: Variation of δ with i (note D_m).

the if the angle of incidence i is varied the angel of deviation δ first decreases,reaches a minimum(D_m) and then increases again.***At the angle of minimum deviation there is only one angle of incidence at all other angles of deviation there are two angles of incidence.***
Thus when

$$\delta = D_m$$

$$i = e \quad and \quad r_1 = r_2 = r$$

Substituting this in (50) we get

$$2r = A \quad or \quad r = \frac{A}{2} \tag{52}$$

since when $delta = D_m$,$e = i$ and applying in (51)

$$D_m = 2i - A \quad or$$

$$i = \frac{D_m + A}{2} \tag{53}$$

Applying Snells law we get

$$\frac{sin\, i}{sin\, r} = n_{21}$$

using equations (53) and (52) we get

$$\frac{sin\,(\frac{D_m+A}{2})}{sin\,\frac{A}{2}} = n_{21} = \frac{n_2}{n_1} \tag{54}$$

This is called the Prism equation.
For a small angle prism, i.e., a thin prism,(A is small) D_m is also very small, and we get by applying the small angle approx to (54) we get

$$n_{21} = \frac{D_m + A}{A} \quad or$$

$$D_m = A(n_{21} - 1) \tag{55}$$

DISPERSION BY A PRISM

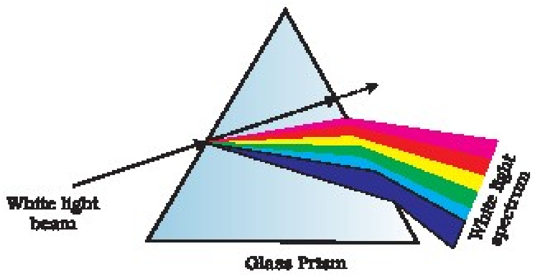


Figure 31: Dispersion of light note:angle of deviation for different colours is different

1. The phenomenon of splitting of light into its component colours is known as dispersion.
2. The pattern of colour components of light **v**iolet, **i**ndigo, **b**lue, **g**reen, **y**ellow, **o**range and **r**ed (VIBGYOR)is called the spectrum of light.
3. The red light bends the least, while the violet light bends the most.

CAUSES OF DISPERSION
Dispersion takes place because the refractive index of medium for different wavelengths (colours) is different.***(wave length is inversely related to wave length)*** For example, the bending of red

component of white light is least while it is most for the violet.Equivalently, red light travels faster than violet light in a glass prism or the refractive index of violet(1.533)in glass is greater than that for red.

NOTE: Not all media are dispersive,for example vacuum or air does not disperse light hence we see sun light as white light.

CHROMATIC ABERRATION: This is a defect in thick lenses. due to their shape thick lenses disperse light producing a coloured halo around the image this is called chromatic aberration.

THE RAINBOW:

- The rainbow is caused by the dispersion of sunlight by the water drops in the atmosphere, due to the combined effect of refraction and reflection of sunlight by spherical water droplets in the atmosphere.
- ***CONDITION FOR THE FORMATION OF RAINBOW:***
  - The conditions for observing a rainbow are that the sun should be shining in one part of the sky (say near western horizon) while it is raining in the opposite part of the sky (say eastern horizon).
  - An observer can observe the rainbow only when his back is toward's the sun.
- **Formation of rainbows:** Consider Fig.32. Sunlight is first refracted as it enters a raindrop, which causes the different wavelengths (colours) of white light to separate.Longer wanve length of light (red) are bent the least while the shorter wavelength (violet) are bent the most. Next, these component rays strike the inner surface of the water drop and get internally reflected if the angle between the refracted ray and normal to the drop surface is greater then the critical angle (48, in this case). The reflected light is refracted again as it comes out of the drop as shown in the figure. It is found that the violet light emerges at an angle of 40 related to the incoming sunlight and red light emerges at an angle of 42. For other colours, angles lie in between these two values.

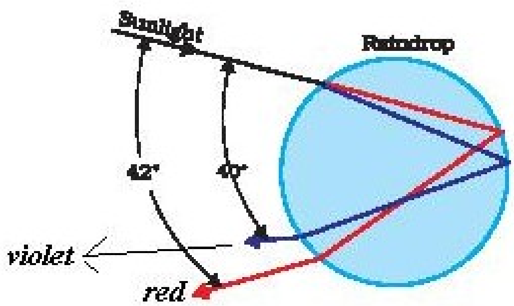


Figure 32: Refraction and total internal Reflection of sun rays in raindrops

- ***Formation of secondary rainbow:***When light rays undergoes two internal reflections inside a raindrop, instead of one as in the primary rainbow, a secondary rainbow is formed as shown in Fig. 9.27(c). It is due to four-step process. The intensity of light is reduced at the second reflection and hence the secondary rainbow is fainter than the primary rainbow. Further, the order of the colours is reversed in it as is clear from Fig.33.

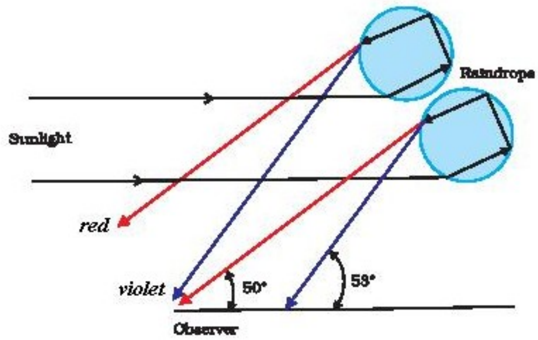


Figure 33: Secondary Rainbow-note the reversal of colours

SCATTERING: The changing of lights direction as it travels through a medium by the particles of a medium is called ***scattering***.

- **The amount of scattering is inversely proportional to the fourth power of the wavelength. This is known as *Rayleigh* scattering.** Light of shorter wavelengths is scattered much more than light of longer wavelengths.
- **BLUISH COlOUR OF THE SKY**:Since blue has a shorter wavelength than red and is scattered much more strongly. In fact, violet gets scattered even more than blue, having a shorter wavelength.But since our eyes are more sensitive to blue than violet, we see the sky blue.
- **Reddish appearance of the sun at sunset or sunrise**, the suns rays have to pass through a larger distance in the atmosphere.Most of the blue and other shorter wavelengths are removed by scattering.The least scattered light reaching our eyes, therefore, the sun looks reddish. This explains the reddish appearance of the sun and.

OPTICAL INSTRUMENTS

THE EYE:
STRUCTURE OF THE EYE: Figure 34 shows the eye. Light enters the eye through a curved front surface, the cornea. It passes through the pupil which is the central hole in the iris. The size of the pupil can change under control of muscles. The light is further focussed by the eye lens on the retina. The retina is a film of nerve fibres covering the curved back surface of the eye. The retina contains rods and cones which sense light intensity and colour,respectively, and transmit electrical signals via the optic nerve to the brain which finally processes this information. ***The shape (curvature) and therefore the focal length of the lens can be modified somewhat by the ciliary muscles.***
KEY points about the eye:

- **Accommodation of the Eye:**When the object is brought closer to the eye, in order to maintain the same image-eye lens distance (about 2.5 cm),The focal length of the eye lens becomes shorter (by changing the curvature of the eye lens)by the action of the ciliary muscles. This property of the eye is called accommodation.
- **Least distance of distinct vision:**When an object is too close to the eye, the lens the lens cannot curve enough to focus the image on to the retina, and the image is blurred. The

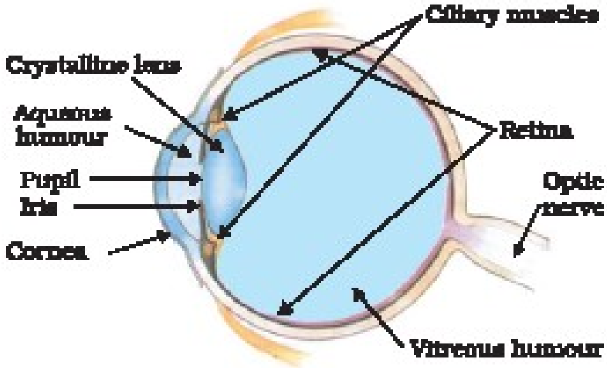


Figure 34: The structure of the eye

closest distance for which the lens can focus light on the retina is called the ***least distance of distinct vision, or the near point***. The standard value for normal vision is taken as 25 cm. (Symbol **D**) **NOTE:** The near point or the *least distance of distinct vision* decreases with age.

EYE DEFECTS

- **Near Sightedness or Myopia:**When light from a distant object arriving at the eye-lens may get converged at a point in front of the retina. This type of defect is called nearsightedness or myopia.
  Symptoms:A person is able to see nearby objects clearly but distant objects appear blurred.

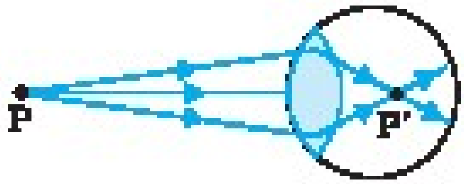


Figure 35: Myopia-Note the image is formed in front of the retina

Cause: This defect is caused when the eye lens is producing too much convergence in the incident beam Fig 35.
Correction: To correct this defect (i.e compensating too much convergence of the eye lens), we interpose a concave lens between the eye and the object, with the diverging effect desired to get the image focused on the retina [Fig.36].

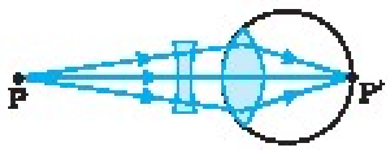


Figure 36: Myopia corrected with concave lens

- **Farsightedness or hypermetropia:**If the eye-lens focusses the incoming light at a point behind the retina then the defect is called farsightedness or hypermetropia.[Fig.37]

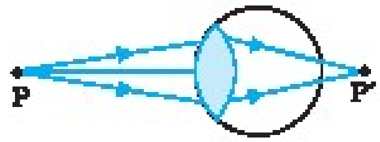


Figure 37: Long sight/hypermetropia-Note the image is formed behind the retina

Symptoms:A person is able to see distant objects clearly but nearby objects appear blurred.
Correction:A convergent lens is needed to compensate for the defect in vision.

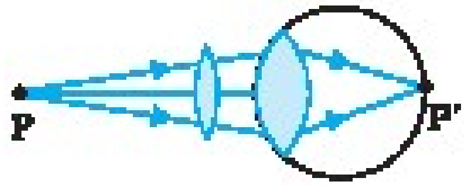


Figure 38: Long sight is corrected with convex lens

- **Astigmatism:**When the cornea is not spherical in shape. For example, the cornea could have a larger curvature in the vertical plane than in the horizontal plane or vice-versa.
Symptons: If a person with such a defect in eye-lens looks at a wire mesh or a grid of lines, focussing in either the vertical or the horizontal plane may not be as sharp as in the other plane. Astigmatism results in lines in one direction being well focussed while those in a perpendicular direction may appear distorted [Fig.39].

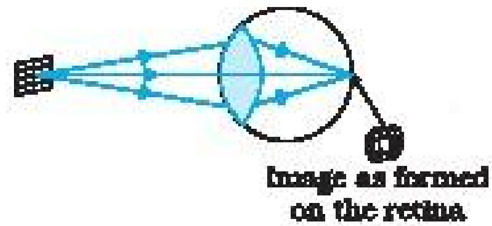


Figure 39: Astigmatism

Correction: Astigmatism can be corrected by using a cylindrical lens of desired radius of curvature with an appropriately directed axis. This defect can occur along with myopia or hypermetropia.

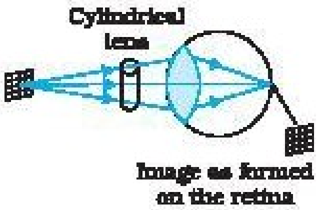


Figure 40: Astigmatism corrected with cylindrical lens

SIMPLE MAGNIFIER or SIMPLE MICROSCOPE or The MAGNIFYING GLASS

What's That?: A simple magnifier or microscope is a converging(that's convex dude) lens of small focal length.

What for is it used?: It is used get an erect, magnified and virtual image of the object at a distance so that it can be viewed comfortably, i.e., at 25 cm or more.

How is it used?: In order to use such a lens(yeah the magnifying glass!) as a microscope, the lens is held near the object, one focal length away or less(remember when the object distance $u \leq f$ enlarged virtual image is formed), and the eye is positioned close to the lens on the other side.

The linear magnification m, for the image formed at the near point(Least Distance of Distinct Vision)D, by a simple microscope

When the object is at a distance slightly less than the focal length of the lens, the image is virtual

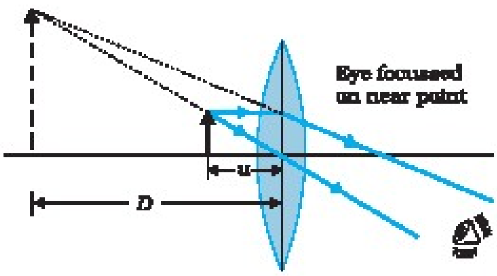


Figure 41: Magnification of a Simple microscope image formed at "D"

and closer than infinity. The closest comfortable distance for viewing the image is when it is at the near point (distance $D = 25cm$).The ray diagram for a simple magnifier is shown below:-

The magnification produced when the Image is formed at the least distance of distinct vision can be calculated as follows:-

The magnification of a lens is given by eqn.(45)

$$m = \frac{v}{u}$$

Using the lens equation (44)

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

hence

$$\frac{1}{u} = \frac{1}{v} - \frac{1}{f}$$

substituting the above equation for $1/u$ in $m = \frac{v}{u}$ we get

$$m = \left(1 - \frac{v}{f}\right)$$

According to sign convention, v is negative, and is equal in magnitude to D. using $v = -D$,we get the magnification as

$$m = \left(1 + \frac{D}{f}\right) \tag{56}$$

The linear magnification m, for the image formed at *infinity*, by a simple microscope If the object is at a distance f, the image is at infinity. When the image is at infinity we have to use the ***angular magnification.***

Angular magnification: Angular magnification m is defined as the ratio of the angle subtended by an object with a lens in use (angle θ in Fig.42b) to the angle subtended by the object placed at the near point with no lens in use (angle θ_0 in Fig.42a):

$$m = \frac{\theta_i}{\theta_0} \tag{57}$$

The ray diagram is shown below:-

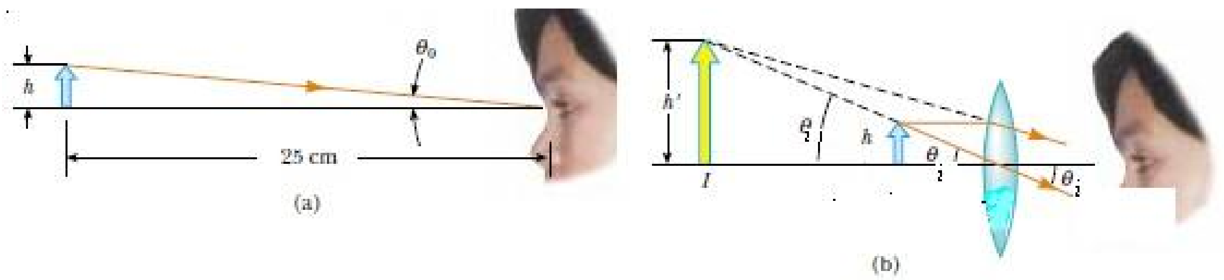


Figure 42: Defnition of angular Magnification

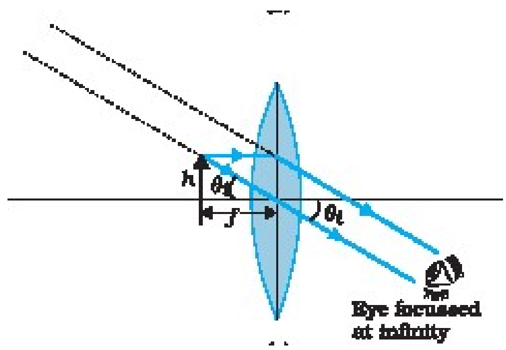


Figure 43: Simple microscope Magnification when the image is formed at infinity.

From the ray diagram fig.43 above

$$\tan\theta_i \approx \theta_i = \frac{h}{f}$$

Similarly from fig.42a

$$\tan\theta_o \approx \theta_i = \frac{h}{D}$$

From the definition of angular magnification m we get

$$m = \frac{\theta_i}{\theta_0} = \frac{\frac{h}{f}}{\frac{h}{D}} = \frac{D}{f} \tag{58}$$

This is one less than the magnification when the image is at the near point(D), Eq. ((56)), but the viewing is more comfortable and the difference in magnification is usually small.Further more when the image is formed at D it causes some strain on the eye Therefore, the image formed at infinity is often considered most suitable for viewing by the relaxed eye.

Compound Microscope.

A simple magnifier provides only limited magnification. Greater magnification can be achieved by combining two lenses in a device called a compound microscope, a schematic diagram of which is shown in Figure .

- It consists of two lenses, the objective(near the object), that has a focal(f_0) a second lens, the eyepiece(near the eye), that has a focal length(f_e).
- The object,is placed just outside the focus of the objective,and forms a real, inverted image $A'B'$,this image is located at(when we want the final image to be formed at infinity) or close(when we want the final image to be formed at "D''") to the focus of the eyepiece(F_e).This Image $A'B'$ serves as the object for the second lens, the eyepiece.
- The eyepiece, which serves as a simple magnifier,produces $A''B''$ a virtual, enlarged image of $A'B'$.
- The final image is inverted with respect to the original object.

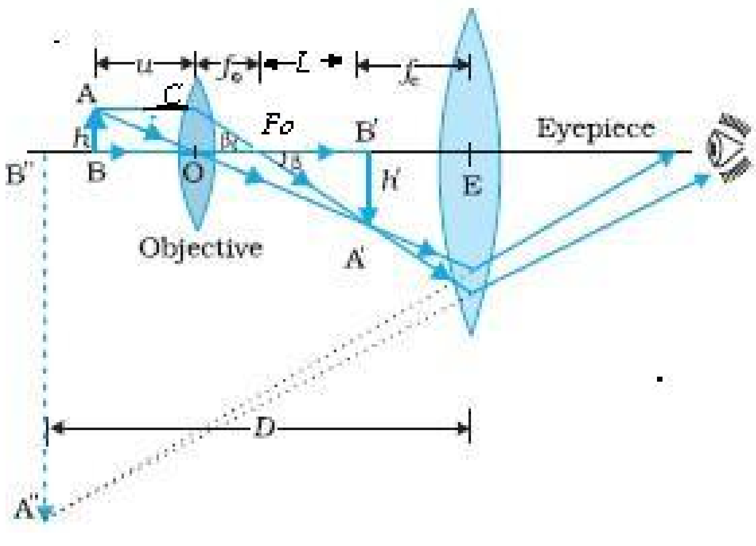


Figure 44: Ray diagram for a compound microscope

Magnification:

The linear magnification m_0 of the objective can be calculated as follows:-

from the ray diagram in $\triangle OCF_0$

$$\tan\beta = \frac{h}{f_0} \tag{59}$$

(note $OC = AB = h$)
in $\triangle OCF_0$

$$\tan\beta = \frac{h'}{L} \tag{60}$$

equating (59) to (**??**)

$$\frac{h'}{L} = \frac{h}{f_0} \tag{61}$$

hence

$$\frac{h'}{h} = m_0 = \frac{L}{f_0}$$

here L is the distance between the second focus of the objective and the first focus the eyepiece is called the tube length of the compound microscope.
As the first inverted image is near the focal point of the eyepiece,and as the eyepiece acts as a simple microscope the magnification of the eye piece is given by eqn.(56)

$$m_e = \left(1 + \frac{D}{f_e}\right)$$

Thus the total magnification when the image is formed at the near point(D) is given by:

$$m = m_0 m_e = \frac{L}{f_0}\left(1 + \frac{D}{f_e}\right) \tag{62}$$

When the image is formed at infinity then the magnification of the eyepiece is given by eqn.(58)

$$m_e = \frac{D}{f_e}$$

Thus, the total magnification , when the image is formed at infinity, is

$$m = m_o m_e = \left(\frac{L}{f_0}\right)\left(\frac{D}{f_e}\right) \tag{63}$$

To achieve a large magnification of a small object (hence the name microscope), the objective and eyepiece should have small focal lengths as bot f_0 and f_e are in the denominator.

Refracting Telescope

- The telescope is used to provide angular magnification of distant objects.
- It has an objective and an eyepiece.
- (Fig. 9.32). It also has an objective and an eyepiece.
- The objective has a ***large focal length*** and a much ***larger aperture*** than the eyepiece.The aperture is made large so as to collect more light from distant objects in the night sky.

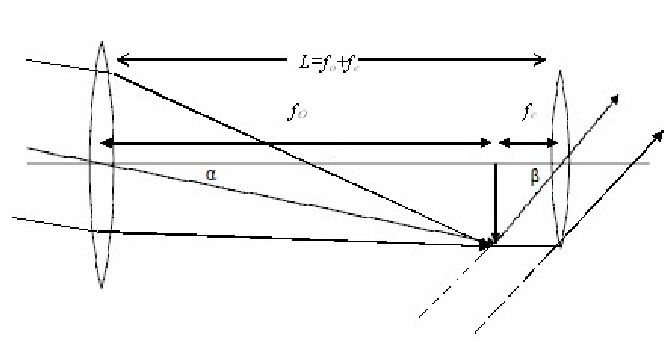


Figure 45: Ray diagram for telescope Image formed at infinity

- **Principle:** Light from a distant object enters the objective and a real image is formed in the tube at its second focal point. The eyepiece magnifies this image producing a final inverted image.

The magnifying power m ***is the ratio of the angle*** β ***subtended at the eye by the final image to the angle*** α ***which the object subtends at the lens or the eye.*** Hence

$$m = \frac{\beta}{\alpha}$$

$$\tan\beta \approx \beta = \frac{h}{f_e}$$

$$\tan\alpha \approx \alpha = \frac{h}{f_0}$$

hence

$$m = \frac{\beta}{\alpha} = \frac{f_0}{f_e} \tag{64}$$

The tube length of the telescope is given by

$$L = f_0 + f_e \tag{65}$$

Terrestrial Telescopes: Terrestrial telescopes have, in addition, a pair of inverting lenses to make the final image erect. Refracting telescopes can be used both for terrestrial and astronomical observations.

Requirements of a Telescope

- The main considerations with an astronomical telescope are its light gathering power and its resolution or resolving power. The former clearly depends on the area of the objective. With larger diameters, fainter objects can be observed.

- The resolving power, or the ability to observe two objects distinctly, which are in very nearly the same direction, also depends on the diameter of the objective. So, the desirable aim in optical telescopes is to make them with objective of large diameter.

Reflecting Telescope(Cassegrain Telescope)

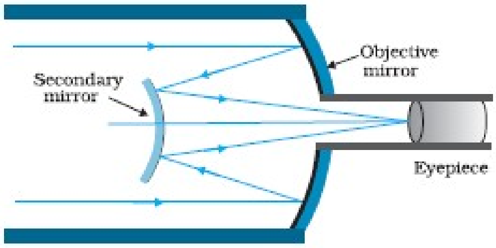


Figure 46: Cassegrain reflecting telescope

Telescopes with mirror objectives are called reflecting telescopes.(Refracting telescopes have lenses)One such arrangement using a convex secondary mirror to focus the incident light,which now passes through a hole in the objective primary mirror, is shown This is known as a Cassegrain telescope, after its inventor.It has the advantages of a large focal length in a short telescope.

Advantages of a Reflecting Telescope over a Refracting Telescope

- Big lenses tend to be very heavy and therefore, difficult to make and support by their edges. Further, it is rather difficult and expensive to make such large sized lenses which form images that are free from any kind of chromatic aberration and distortions.
- There is no chromatic aberration in a mirror.
- If a parabolic reflecting surface is chosen, spherical aberration is also removed.
- Mechanical support is much less of a problem since a mirror weighs much less than a lens of equivalent optical quality, and can be supported over its entire back surface, not just over its rim.

End